On Thermal Boundary Layer Growth on Continuous Moving Belts

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Sakiadis (*1* to *3*) was the first investigator to examine in detail the mathematical equations describing transport phenomena from a continuous moving belt. As Sakiadis pointed out and other investigators have confirmed (*4* to *8*), the parabolic momentum transport equation can be used as a mathematical model if the boundary conditions reflect the fact that the belt surface is moving and infinite in length.

Griffith (*4*) and Erickson et al. (*5*) have examined mass, heat, and momentum transport, assuming similar boundary conditions of constant belt concentration and isothermal belt temperature. Tsou et al. (*6, 7*) have examined the stability of the boundary layer on this type of surface, both experimentally and theoretically.

The first experimental work done (excluding a photograph without explanation, given by Sakiadis) was on momentum transport from a moving cylinder, by Griffith (*5*). Griffith restricted his system to linear velocities of 0.1 to 2.2 ft./sec. and his observations to the region 2 to 20 in. from the slot. His results indicated that the velocity gradients were consistently greater than those predicted by Sakiadis and the deviation increased with distance from the slot and decreased with belt speed. Griffith argued, however, that the physical bounds on his experiment may have caused the measurements to be consistently in error.

Sano and Yamada (*8*) reexamined Griffith's work, determining heat transfer coefficients from spun filaments. Their agreement with Sakiadis's theory was quite good.

Tsou et al. (*6, 7*) developed a unique method for creating the continuous surface, using a heated rotating drum of large diameter. From thermocouples embedded in the surface of the drum and suspended in the ambient air above the drum surface, they were able not only to measure heat transfer coefficients, but to determine laminar-to-turbulent transition regions for both momentum and heat transport. It should be noted that the apparent belt velocity range for Tsou et al. was 35 to 80 ft./sec.

In a contemporary study, we have attempted to observe experimentally the *thermal* boundary layer growth from a passive, flat belt, using the traditional Schmidt shadowgraph method (*10*). The equipment, the optical setup and its attendant theory, and the experimental procedure are described elsewhere (*9*). In this note, we wish to compare our results with those predicted by Erickson et al. for the isothermal belt.

Our belt temperature was held at 175°F., and was operated at one of the four speeds given in the box on Figure 1 into ambient air at 75°F. From boundary layer theory, it is predicted that the heat transfer coefficient is proportional to the reciprocal square root of the distance downstream from the slot and directly proportional to the square root of the belt speed. This theory is presented by the solid lines in Figure 1, together with our experimental data. The Colburn-type equation

$$N_{Nu_x} = CN_{Re_x}{}^{a}\ N_{Pr}{}^{b} \qquad (1)$$

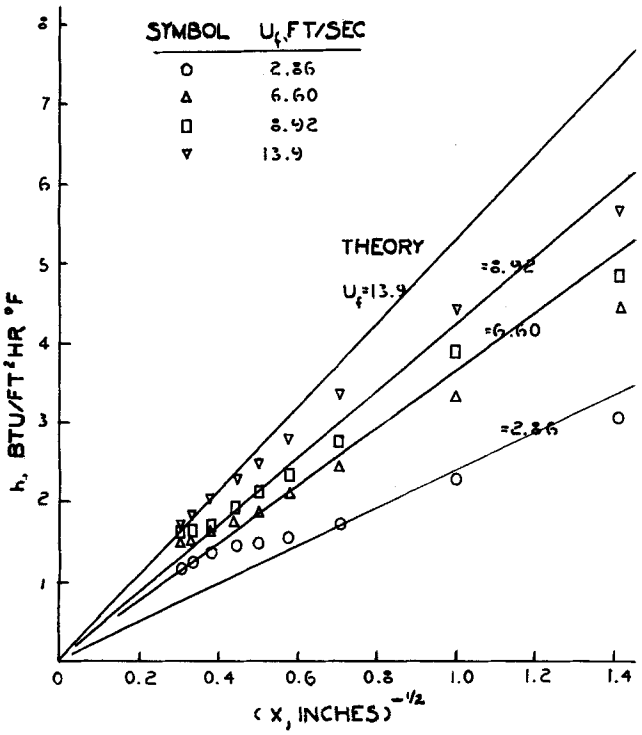

Fig. 1. Thermal boundary layer growth on a continuous flat surface: comparison of theoretical and experimental results. h vs. $x^{-1/2}$ with belt velocity as a parameter.

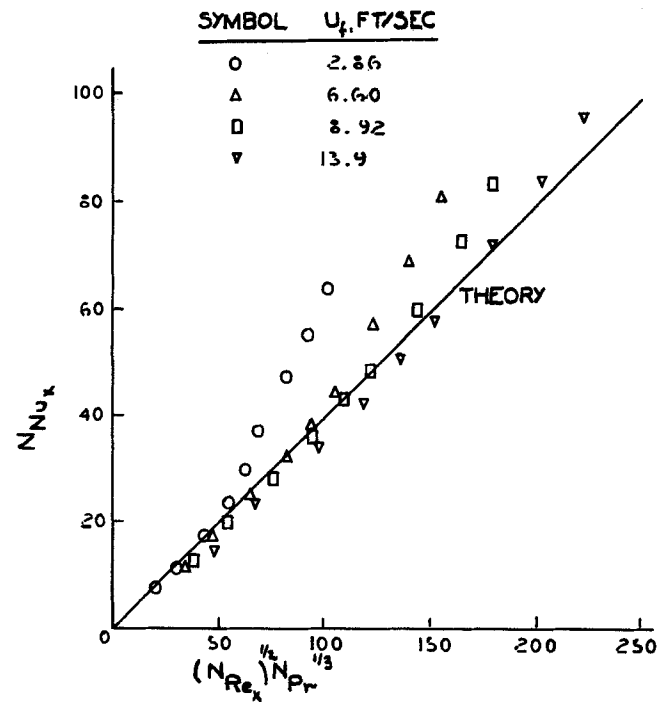

Fig. 2. Thermal boundary layer growth on a continuous flat surface: comparison of theoretical and experimental results. Nusselt number as a function of Reynolds and Pandtl numbers.

where $N_{Nu_x} = hx/k$, the Nusselt number based on the distance from the slot, $N_{Re_x} = xU_f\rho/\mu$, the Reynolds number based on the distance from the slot and the belt speed, and $N_{Pr} = C_p\,\mu/k$, which we assumed to be a constant for our sets of experiments, is shown in Figure 2 as a solid line, along with our data.

We calculated the belt temperature drop from the slot to the return slot from an overall heat balance and from the method suggested in the appendix of Griffith's paper and found it to be on the order of 2 to 5°F. for our set of runs, supporting our assumption of isothermal belt temperature.

We found that in the region of ½ in. from the slot, the experimental heat transfer coefficients were 10 to 25% lower than those predicted by the Erickson theory. We attribute this to the presence of a vertical wall and the corresponding movement of air down the wall and from the edges of the belt, replacing that which has been dragged along by the belt. The analysis of this three-dimensional boundary layer in this region was not attempted. At a distance approaching one ft. from the slot, the experimental values were 10 to 60% higher than those predicted. We attribute this to natural convection heat transfer. Increasing belt speed caused an expected decrease in the natural convection effect and an expected increase in the wall effect.

It appears that, within experimental error discussed above, we are in good agreement with the theoretical prediction with the theory of Erickson et al.

Further studies using continuously extruded sheet and a laser grating interferometer are in progress.

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NOTATION

C_p = specific heat, air, B.t.u./lb. °F.
h = forced convection heat transfer coefficient, B.t.u./sq.ft. °F.
k = thermal conductivity, air, B.t.u./hr. sq.ft. °F./ft.
U_f = belt velocity, ft./sec.
x = distance along belt, from slot, in.
μ = viscosity, air, centipoise
ρ = density, air, lb./cu.ft.
N_{Nu_x} = Nusselt number, based on belt distance from slot, dimensionless
N_{Re_x} = Reynolds number, based on belt distance from slot and belt velocity, dimensionless
N_{Pr} = Prandtl number, for air, dimensionless

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Use of Time Delays in Packed Gas Absorption Column Simulation

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There have been relatively few studies of the dynamic behavior of packed absorption or distillation columns. This is probably because such continuous systems involve more than one space variable and have a complex geometry. For these reasons it is difficult to formulate a meaningful theoretical transfer function for packed columns. However, two very different approaches to the analysis of such distributed parameter systems are already evident (8). These are (*a*) successively more rigorous partial differential equation models that attempt to explain the dynamic behavior by a detailed analysis of the mechanisms involved, or (*b*) relatively simple, highly approximate ordinary differential equation models that are suitable for control purposes. The second approach has been applied to parallel simulation and other feedforward control schemes that require a simple, approximate process model (8). This paper, which is concerned with the second approach, shows that the inclusion of time delay elements in a mixing-cell model gives a simpler analog computer model than previously reported.

If we neglect radial concentration changes, the two-phase packed absorption column can be represented by partial differential equations involving only time and one space variable as independent variables (3). By applying finite difference techniques, these partial differential mass balances for each phase can be reduced to a set of ordinary difference equations. Since these are similar to the mass balances of staged process equipment, it is logical to use a staged mixing-cell model to represent the set of ordinary difference equations. Gray (3, 4) used a staged CSTR model consisting of mixing cells in series for each phase. The component mass balances used by Gray were

$$x_{n+1}\,L - x_nL + RaH = Hh_L\frac{dx_n}{dt} \qquad (1)$$

for the liquid-phase cells and

$$y_{n+1}\,G - y_nG - RaH = Hh_G\frac{dy_n}{dt} \qquad (2)$$

for the vapor-phase cells.